

# Adaptive Virtual Model Control of a Bipedal Walking Robot

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## Abstract

*The robustness of bipedal walking robots can be enhanced by the use of adaptive control techniques. In this paper, we extend a previous control approach, "Virtual Model Control" (VMC) [6] to create "Adaptive Virtual Model Control" (AVMC). The adaptation compensates for external disturbances and unmodelled dynamics, enhancing robustness in the control of height, pitch, and forward speed. The state machine used to modulate the virtual model components and to select the appropriate virtual to physical transformations (as in traditional VMC) is also used to inform the adaptation about the robot's changing configuration.*

*The design procedure for AVMC is described in this paper and simulation results are presented for a planer walking biped.*

**Keywords:** *adaptive control, virtual model control, legged locomotion.*

## 1 Introduction

Because a biped's dynamics are multivariable, high-order, nonlinear, and time-variant, it is difficult to design a walking controller using traditional techniques. However, many novel approaches have been explored.

Golliday and Hemarni [2] used state feedback to decouple the high-order system of a biped into independent low-order subsystems. Miyazaki and Arimoto [10] used a singular perturbation technique and showed that bipedal locomotion can be divided into two modes: a fast mode and a slow mode, thus simplifying the controller's design. Furusho and Masubuchi [11] derived a reduced order model as a dominant subsystem that approximates the original high-order model very well by applying local feedback control to each joint of a biped robot. Miura and Shimoyama [7] linearized biped dynamics and designed stable controllers by means of linear feedback. Kajita and Tani [12] developed their 6-degree of freedom bipedal robot "Meltran II" using a linear Inverted Pendulum Mode successfully. A research group at the Honda Motor Company designed their control system for a humanoid bipedal robot using zero moment force control and playback of recorded trajectories [5]. In addition, other researchers have made good progresses in the control of

biped robots by means of learning techniques such as fuzzy logic and neural network control [13,15].

In our previous "Virtual Model Control" approach to bipedal walking [4,6], we utilized physical intuition in the development and implementation of control strategies. With this approach we achieved moderate performance and robustness for blind planer walking over rough terrain.

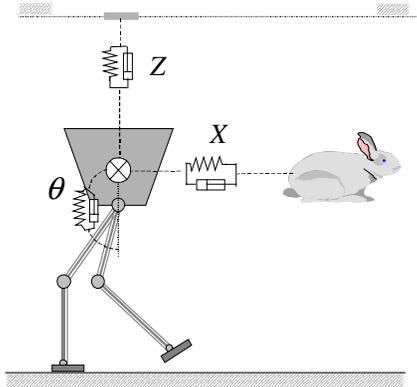
In this paper, we add adaptation to the Virtual Model Control approach to enhance robustness. Simulation results are presented in section 4.

## 2 Virtual Model Control

Dynamically stable legged robots are difficult to control for several reasons. They are non-linear, passively unstable, under-actuated, and exhibit varying dynamics depending on which feet contact the ground. Because of these difficulties, textbook control solutions typically are not applicable. Instead, physical intuition is often used as a tool to develop a controller.

Virtual Model Control [4,6,16,17] is one such technique. Virtual components are attached between parts of the physical structure of the robot and between the robot and the environment. Torque is applied to the joints of the robot so as to make the robot behave as if the virtual components are present. A finite state machine monitors the robot's configuration and discretely modulates the virtual to physical transformation and the parameters of the virtual components.

Figure 1 shows a diagram of one set of virtual components that can be used to control a planar bipedal walking robot. These components were used in the control of our 4-DOF walking robot Spring Turkey [6]. Virtual spring-damper components are attached to the robot in three axes ( $Z$ ,  $X$ ,  $\theta$ ), and provide height, pitch, and forward velocity control. The "dogtrack bunny" indicates that a spring-damper mechanism in the  $X$  direction is pulled along at the desired velocity. Due to the constraint of an un-actuated ankle in this robot, the  $X$  axis spring-damper mechanism is attached only when the robot is in its double support phase of walking.



**Figure 1:** One implementation of Virtual Model Control applied to a seven-link bipedal walking robot.

There are three steps to implementing a Virtual Model Controller:

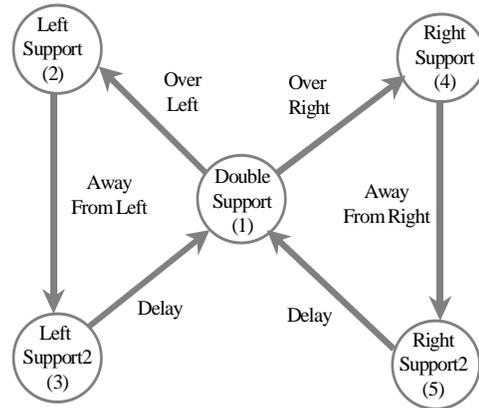
- 1) Design the controller by choosing virtual components and their attachment points.
- 2) Design the finite state machine or other method of virtual component modulation.
- 3) Determine the virtual to physical transformation.

Figure 2 shows a state machine that was used in the control of our bipedal robots “Spring Turkey” and, later, “Spring Flamingo” (which had actuated ankles). The virtual to physical transformation is based on the robot’s Jacobian and some additional constraints [6].

### 3 Adaptive Control in Virtual Dynamics Space

Virtual Model Control without adaptive mechanisms can control a walking robot successfully over both level and sloped terrain [16,17]. However, it is beneficial to consider the higher order unmodelled dynamics of the robot and dynamically adapt to changing dynamics or disturbances. Chew [16] used robust adaptive control with mass adaptation in his simulation study of rough terrain walking. We extend Chew’s work here into a more complete virtual dynamics space framework.

As a natural extension of VMC, Adaptive VMC can also be considered as a learning mechanism. A good learning mechanism needs a proper performance index function, a learning algorithm, and a suitable system dynamics framework. In the following sub-sections, the above three components are discussed. A virtual dynamics model based adaptive control framework is presented.

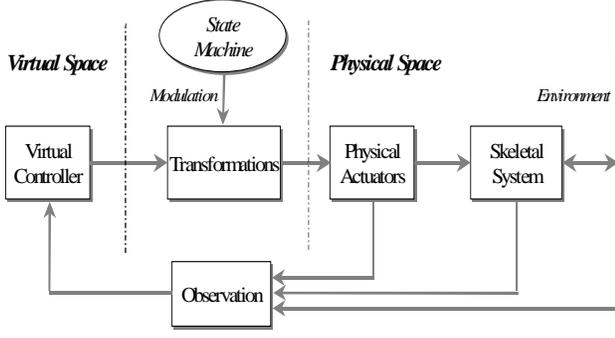


**Figure 2:** Diagram of the global state machine for a bipedal walking robot.

### 3.1 Performance index of the control in a legged robot

Robustness for a walking robot requires that its stability be maintained when encountering unexpected external disturbances and complex environments. There are two types of stability for a legged robot. First, stability of a legged robot requires the stability of internal dynamics in each individual mode under external disturbances. Second, achieving stability also requires persistent smooth movements in complex environments. In this paper, the adaptive VMC will take care of the first requirement for stability. The second requirement can be achieved through an adaptive gait control approach. We will not focus on that in this paper.

In general, the performance measurements of walking robots are much different from the typical notions of performance for manipulators, such as command following and disturbance rejection. The overall performance of a walking robot is usually defined in terms of biological similarity and efficiency of leg locomotion, smoothness of movement and robustness to the environments. Specifying a proper overall performance index for a legged robot is thus a difficult task. In this preliminary work, our approach is to utilize a simpler tracking index and try to achieve dynamic stability in a Lyapunov sense.



**Figure 3:** Diagram of virtual dynamics model based control design.

### 3.2 Virtual dynamics model based control design approach

Automatic tuning of a controller requires a suitable framework or a dynamic model that can reflect the physical interaction between the environment and the system itself. In this paper, our adaptive control design is based on the framework of a virtual dynamics model. Figure 3 shows the diagram of a virtual dynamics model based control design. In this diagram, there is a virtual dynamics space and a physical dynamics space. By utilizing an observation module, the necessary information is collected from the physical space and formulated into the properly selected virtual axes of the virtual space, such that the virtual dynamics of the biped robot can be reconstructed. The virtual control is designed based on the reconstructed virtual dynamics model. The outputs of the controller are the generalized virtual forces that are transformed into the physical torque commands for the actuators by means of the dynamics transformations. The transformations are different in different states, for instance, the single support states and the double support state.

In general, the control law of a dynamic system can be formulated as,

$$u = -K_d(\dot{\tilde{x}} + \lambda\tilde{x}) + \Delta u_c \quad (1)$$

where the control is composed of a linear feedback control part plus a control action correction term  $\Delta u_c$ ,  $\lambda, K_d > 0$ , and  $\tilde{x}$  is system tracking error. Here we call  $\Delta u_c$  a learning control term which will be updated on line in an adaptive control system. How to determine  $\Delta u_c$  is the focus of this paper. Our approach is to utilize the information observed from physical space and compute

$\Delta u_c$  based on the reconstructed virtual model in virtual space.

The formulation of the virtual dynamics is based on the concept of linearization of dynamics, which says that any nonlinear dynamics equation can be linearized into a locally linear dynamics equation around an operating point (state) and globally the dynamics can be considered as time-varying linear dynamics. Therefore, in our design, we use the form of a time-varying linear virtual dynamics model and put the error model (unmodelled dynamics) into an error bound for a robustness mechanism to tolerate. For simplicity, in this design, a second order virtual system model is utilized. It is expected that the controller designed in virtual space should be able to take care of the unmodelled dynamics. We choose the adaptive sliding control approach with dead zone to handle this problem.

In our design, the dynamics of the biped legged robot is formulated in  $z$ ,  $x$ , and  $\theta$  axis of the virtual space. The general form of the virtual model in ( $z$ ,  $x$ , and  $\theta$  axis) can be written as,

$$a_1\ddot{x} + a_2\dot{x} + a_3x + a_4 + f_x(x, \dot{x}, t, \dots) + d_x = u_x \quad (2)$$

$$b_1\ddot{z} + b_2\dot{z} + b_3z + b_4 + f_z(z, \dot{z}, t, \dots) + d_z = u_z \quad (3)$$

$$c_1\ddot{\theta} + c_2\dot{\theta} + c_3\theta + c_4 + f_\theta(\theta, \dot{\theta}, t, \dots) + d_\theta = u_\theta \quad (4)$$

where  $x$ ,  $z$ ,  $\theta$  are the state variables,  $u_x, u_z, u_\theta$  are the control commands and  $f_x(x, \dot{x}, t, \dots)$ ,  $f_z(z, \dot{z}, t, \dots)$ ,  $f_\theta(\theta, \dot{\theta}, t, \dots)$  are the unmodelled dynamics terms which are unknown functions of the state variable  $x, \dot{x}, z, \dot{z}, \theta, \dot{\theta}$  time  $t$ , and the variables,  $d_x, d_z, d_\theta$  are the disturbance terms. The linear crossover terms are not included here in the above equations, but in a general case, they should be present.

### 3.3 Adaptive control design

Using the above virtual dynamics formulation and the framework of virtual dynamics model based control, the adaptive controller can be designed in the virtual space by means of adaptive sliding control theory [9]. Since the linear dynamics of  $z$ ,  $x$ , and  $\theta$  axis are in a similar formulation, the general dynamics (5) of only one axis is described in the following section.

$$a_1\ddot{x} + a_2\dot{x} + a_3x + a_4 + f(x, \dot{x}, t, \dots) + d = u \quad (5)$$

Define the switching variable  $s(t)$  as,

$$s(t) = \dot{\tilde{x}} + \lambda \tilde{x} = \dot{x} - \dot{x}_d \quad (6)$$

where  $\tilde{x} = x - x_d$ ,  $x_d$  is the desired trajectory,  $\lambda$  is a strictly positive gain (except  $\lambda=0$  for  $x$  axis dynamics).

Note that  $\dot{x}_r$  can be computed from the state  $(x, \dot{x})$  and the desired trajectory  $x_d$ ,

$$\dot{x}_r = \dot{x} - s(t) = \dot{x}_d - \lambda(x - x_d) \quad (7)$$

According to adaptive control theory [9], we can derive the following control law,

$$u = Y\hat{a} - K_D s \quad (8)$$

Choose the adaptation law as,

$$\dot{\hat{a}} = -\Gamma Y^T S_\Delta \quad (9)$$

$$S_\Delta = s - \Phi \text{sat}\left(\frac{s}{\Phi}\right) \quad (10)$$

$$\text{sat}(x) = \begin{cases} x & |x| < 1 \\ \text{sgn}(x) & \text{else} \end{cases} \quad (11)$$

where  $\Phi = \frac{D}{K_D}$ ,  $|d| + |f(x, \dot{x}, t, \dots)| \leq D$ ,  $D$  is the upper

bound of the disturbance and the unmodelled dynamics.  $\hat{a}$  is estimation of the parameter vector  $a$ .  $\Gamma = \text{diag}\{\gamma_1, \gamma_2, \gamma_3, \gamma_4\}$ , ( $\gamma_i > 0$ ) is the adaptation gain matrix.

$$Y = [\ddot{x}_r \quad \dot{x}_r \quad x \quad 1] \quad (12)$$

$$a = [a_1 \quad a_2 \quad a_3 \quad a_4]^T \quad (13)$$

By the above control law and adaptation law, it can be guaranteed that the positive semidefinite Lyapunov function candidate

$$V = \frac{a_1}{2} S_\Delta^2 + \frac{1}{2} \tilde{a}^T \Gamma^{-1} \tilde{a} \quad (14)$$

where  $\tilde{a} = \hat{a} - a$  has a negative semidefinite time derivative. Therefore

$$\dot{V} \leq -K_D S_\Delta^2 \quad (15)$$

From the above result, we can prove uniform global stability of the system ( $S_\Delta \rightarrow 0$ ,  $t \rightarrow \infty$ ) by Barbalat's Lemma [9].

The above adaptive control design can be used to design the adaptive control for  $x$ ,  $z$ ,  $\theta$  axis. Thus the corresponding controls are the force commands  $(f_x, f_z, f_\theta)$  generated in the virtual space. Then following the steps in section 2, the actuator torques can be obtained by forward dynamics transformations. Referring to the general form of control law (1), in this case  $\Delta u_c = Y\hat{a}$ .

The result in equation (15) presents good behavior of asymptotically global stability outside of the sliding boundary layer assuming the given continuous dynamics as in (5). In fact, in our application, the equivalent virtual dynamics in the  $X$ -axis is not a continuous function in terms of the alternate states during bipedal walking. So the performance in  $X$ -axis dynamics is not guaranteed. This is addressed in the conclusions.

It is worthwhile to mention that the above adaptive control scheme has a nice property, namely robustness, which is achieved by means of boundary layer tolerance. The model error and disturbance are all formulated into a pre-estimated error bound. Then the boundary layer thickness is determined based on it. This implies that by adding better identification mechanisms to the above dynamics framework (Figure 3), the performance could be improved further. For example, we can incorporate some nonlinear identification schemes (such as neural networks) to the adaptive control system. Combining a nonlinear identification model such as radial base function neural networks and the above linear time varying model in (5), we can derive the following virtual dynamics equations:

$$a_1 \ddot{x} + a_2 \dot{x} + a_3 x + a_4 + \sum_{i=1}^N C_i g_i(\bullet) + \varepsilon + d = u \quad (16)$$

In (16), we have a mixed model with linear and nonlinear part where  $\varepsilon$  is the model error,  $g_i(\bullet)$  is the nonlinear base functions and  $d$  is the disturbance term. By using this formulation and proper identification techniques, such as neural networks, the model error can be reduced. In this case (not discussed here), the error bound is smaller and the tracking performance further improved.

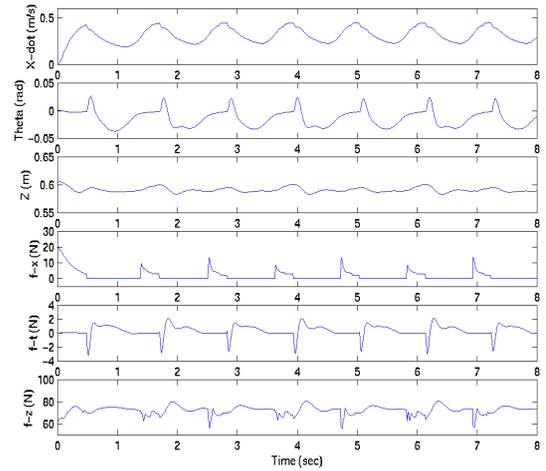
#### 4. Simulation and Results

A planar bipedal walking robot was created in simulation. The simulated biped has a mass of approximately 8.0 kg and stands 0.80 m tall. Both virtual model control [6] and Adaptive Virtual Model Control were applied to the biped. During the simulations, external force disturbances were exerted on the biped in

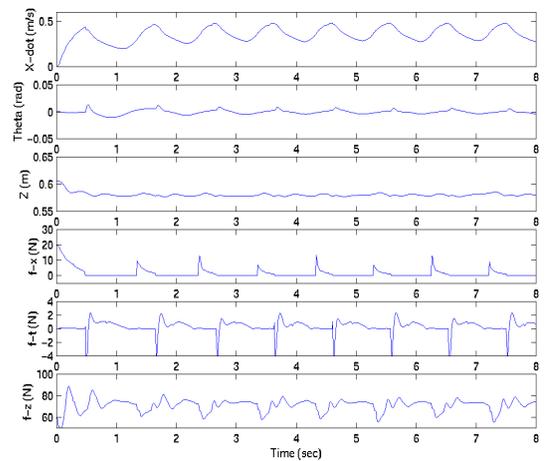
different directions to test the control robustness. We observed that the adaptive virtual model controller improved the system's robustness. When an impulse external force was exerted on the robot, the robot was able to maintain stable walking and recovered its continuous motion. Also, the simulations showed that the biped with AVMC could better maintain the desired height of center of mass (CM), the desired body pitch as well as smooth motion in the x-axis. The simulated biped with AVMC can walk indefinitely.

Figure 4b shows the simulation results with adaptive VMC, the dynamics of force signals  $f_x, f_z, f_\theta$ , and forward velocity in  $x$ , and actual position in  $z, \theta$  (i.e. height, pitch) in virtual space. Figure 4a shows the planar bipedal robot controlled by the VMC scheme. In Figure 4a & 4b, the responses are robot height ( $Z$ ), pitch ( $\theta$ ), forward velocity ( $X\text{-dot}$ ), as well as virtual force commands generated by a controller, such as  $f_z$  ( $f\text{-}z$ ),  $f_x$  ( $f\text{-}x$ ) and  $f_\theta$  ( $f\text{-}t$ ). Comparing the results under VMC and AVMC, we can see that AVMC can improve the dynamic tracking performance of height and pitch, but it can't help much in forward velocity control because the controller can only function in the double support state. In single support states, because of the dynamics constraints, the controller of  $X$ -axis is disconnected by the dynamics transformations. This implies that the virtual dynamics in  $X$ -axis is not a continuous function. Therefore the adaptive control can not really achieve a desired performance in the forward speed control (of  $X$ -axis), This could be further improved by a gait control scheme. Figure 5 shows the parameter identification of the virtual linear dynamics model by the adaptation mechanism.

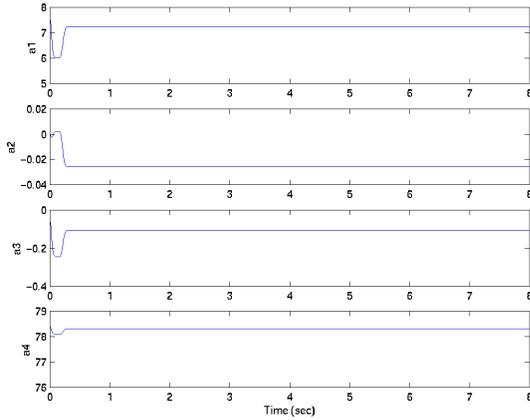
In the test of robustness with external disturbances, we did an external force impact test in our simulation. Figure 6 shows the simulation responses of a bipedal walking robot experiencing an external force impact (10 Newtons) in the  $z$ -direction. Figure 7 shows the stick plot diagram of this walking profile with a force impact. From the above results, it has been shown that improved robustness can be achieved by means of the adaptive VMC scheme. The robustness of the biped with changing terrain will be tested in our future research.



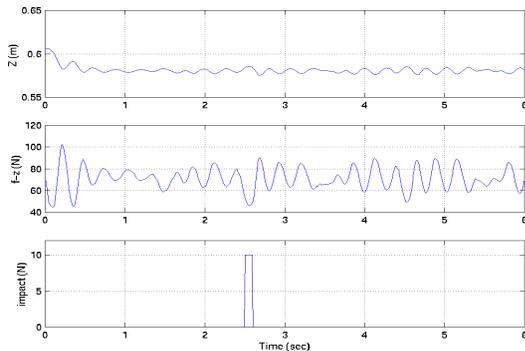
**Figure 4a:** Simulation results of a bipedal robot with VMC.  $X\text{-dot}$ ,  $\theta$ , and  $Z$  are responses of the forward velocity, pitch angle and robot body height respectively.  $f_x$ ,  $f_t$ ,  $f_z$  are the virtual force commands generated by the VMC.



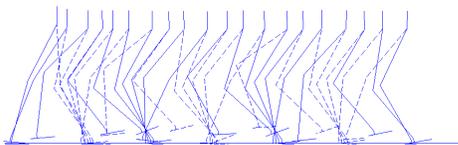
**Figure 4b:** Simulation results of a bipedal robot with Adaptive VMC.  $X\text{-dot}$ ,  $\theta$ , and  $Z$  are responses of the forward velocity, pitch angle and robot body height respectively.  $f_x$ ,  $f_t$ ,  $f_z$  are the virtual force commands generated by the Adaptive VMC.



**Figure 5:** Parameter identification of the linear virtual dynamics model in z-axis.



**Figure 6:** Simulation results of a bipedal walking robot with external impact (10 Newtons) in z direction. Z and f-z are the robot height and the virtual force command in z axis respectively.



**Figure 7:** Stickplot of a bipedal walking robot experiencing external force impact (10 Newtons) in z direction.

## 5 Conclusions

Adaptive Virtual Model Control has been proposed to enhance the robustness of the control system for a bipedal walking robot. When adaptation is added to the virtual components, the controller responds to time varying parameters and external disturbances. It also adapts to unmodelled dynamics, resulting in more accurate height and pitch trajectory tracking.

Under-actuation is a major aspect of dynamic legged robots which makes their control a challenge. Due to the limitations of the foot-ground contact, it is impossible to stabilize a biped under all circumstances. In this study, we assumed the feet and ankles were unactuated and hence chose to control forward velocity only during double support. Thus the stability results of section 3 hold only for pitch and height during single support, and for speed assuming the swing leg and support transition controllers successfully carry the robot from one double support phase to another. Forward speed control could be accomplished by a proper gait control scheme.

It may be possible to extend Virtual Model Control by adding learning components as well as adaptive components. We are currently pursuing this idea.

## Acknowledgement

The first author appreciates his many constructive discussions with J.J. Slotine about the theory underlying this work.

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